LATWAK: IMPACT TEST TO OBTAIN PILE LATERAL STATIC STIFFNESS

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ABSTRACT: The LATWAK test consists of hitting in the horizontal direction the side of a pile on which a horizontal velocity transducer is attached. The blow is delivered with a sledge hammer equipped with a dynamic force transducer. The force time signal from the hammer (input) and the velocity time signal from the pile (output) are recorded during the impact. The experimental mobility curve is obtained as a function of frequency by calculating the modulus of the complex valued ratio of velocity over force using Discrete Fourier Transforms. Theoretically it is assumed that the pile is an elastic member with mass and that the soil can be represented by linear springs and viscous damping. The problem of the steady state forced vibration of the pile in such a soil is solved mathematically. It leads to the theoretical mobility curve for the pile-soil system. The experimental mobility curve obtained in the LATWAK test on the pile is matched with the theoretical mobility curve. A system identification technique is used to match the two curves and to extract the best-fit model parameters, which include the static lateral stiffness K for the pile-soil assembly. To evaluate the usefulness of the method, the lateral stiffness K_p predicted by the LATWAK test on a pile was compared to the lateral stiffness K_m measured in a static lateral load test on the same pile. A total of 20 pile load tests and 20 LATWAK tests were performed and used to compare K_p and K_m. The results are encouraging.

IDEA

The idea of the LATWAK test comes from different sources. It comes in part from the development of the WAK test by Briaud and Lepert (1990). The WAK test or Wave Activated Stiffness (K) test is an impact test to obtain the static stiffness of a spread footing by hitting it with a sledge hammer in the vertical direction while recording the force-time signal from the hammer dynamic load cell (input) and the velocity-time signal from the geophones on the spread footing (output). The LATWAK test comes also from the Nondestructive Techniques (NDT) developed to obtain the axial stiffness of a pile and the influence of defects on that stiffness (Paquet 1968). The idea also finds its basis and background in the work of several-leading researchers in soil dynamics. This includes the work of Novak (1974), Roesset (1980), and Nogami (1988) among others.

While a defect which would reduce the effective diameter of a drilled shaft is definitely undesirable, the axial stiffness is likely to be less affected than the lateral stiffness. Indeed, the axial stiffness is influenced by the square of the diameter (area) while the lateral stiffness is influenced by the fourth power of the diameter (moment of inertia). Therefore the behavior in the lateral direction may be critical. The LATWAK test or Lateral Wave Activated Stiffness (K) test was developed to obtain the lateral static stiffness of a pile in place. This LATWAK stiffness is measured at the small strain level generated by the sledge hammer impact. The stiffness K is defined here as the ratio of the lateral load applied in a static lateral load test over the corresponding lateral displacement; K is in MN/m, for example.

LATWAK TEST

The LATWAK test method consists of hitting a pile horizontally with a sledge hammer, measuring the force-time impulse of the hammer, measuring the horizontal velocity of the pile and analyzing their interaction. Fig. 1 shows a typical setup and test. The sledge hammer weighs about 150 N, and is equipped with a dynamic load cell and a soft rubber tip. The soft rubber tip softens the blow, concentrating the impact energy of the hammer in the low frequency range; this is desirable to be closer to the static phenomenon and to minimize the rate effects. The horizontal impact usually generates between 15 and 50 kN of peak dynamic force. A geophone is placed on the pile vertical face on the opposite side of the point of impact in order to record the horizontal velocity of the pile head during and after the horizontal impact. The peak velocities typically range from 0.02 m/s to 0.2 m/s and the velocity signal damps out after a time generally varying between 0.1 to 0.5 s. An example of the signals collected during a LATWAK test is shown in Fig. 2.

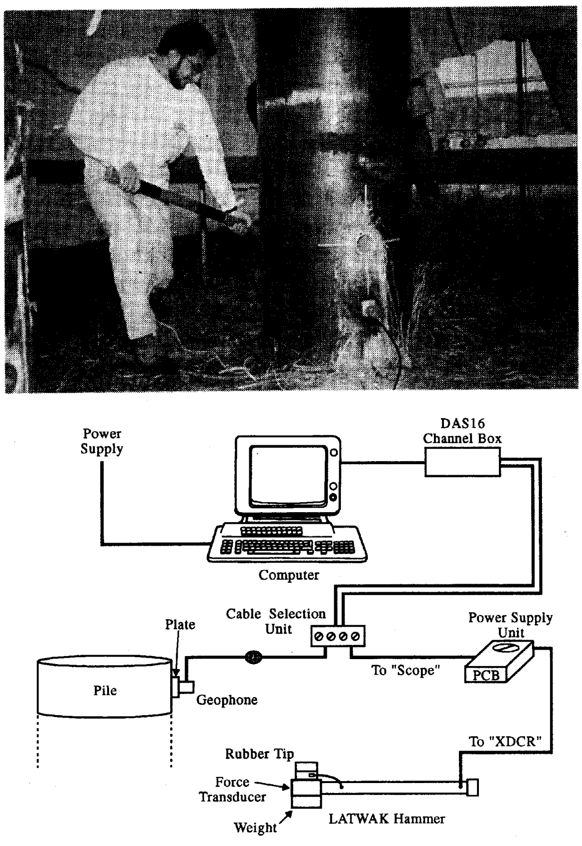

FIG. 1. LATWAK Test

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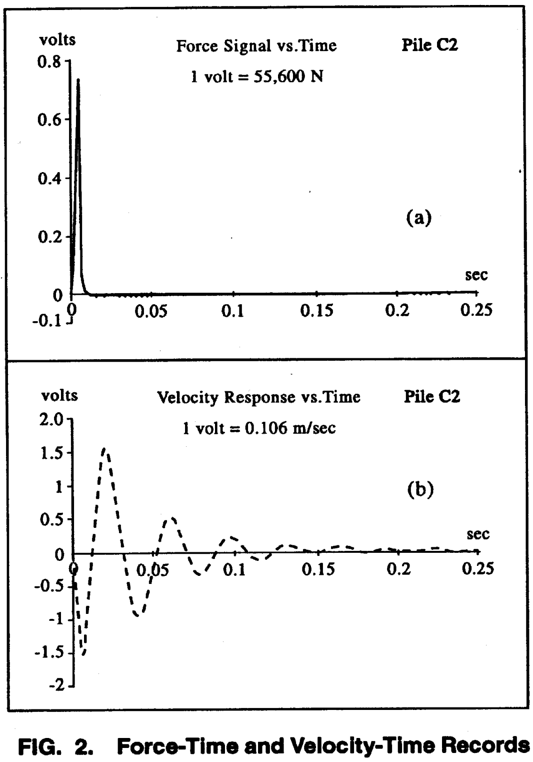

FIG. 2. Force-Time and Velocity-Time Records

THEORETICAL FRAMEWORK

The problem of flexural wave propagation in piles is not a simple matter and one which is much more complicated than the axial stress wave propagation problem. Continuum models, finite element models and discrete models have been used to simulate this soil-pile dynamic interaction. With the continuum models, the soil mass is represented as a continuum with linear elastic or visco elastic properties (Tajimi 1969; Novak 1974; Novak and Nogami 1977). The finite element models offer great flexibility in representing complex geometries and variations in soil properties (Novak 1977; Kuhlemeyer 1979; Dobry et al. 1981). With the discrete models, the soil-pile system is represented by a set of discrete masses, springs and dashpots (Agarwal 1973; Prakash and Chandrasekaran 1973). The following derivation is a variation of the discrete model method. Here, reference is made to Ballouz and Briaud (1993) for the details of the theory because the writers know that all the details of the derivation can be found in it.

The soil is modeled as a set of springs and dashpots (Fig. 3); the spring constant k is constant with depth z and with deflection y, and so is the viscous constant c. The spring constant k per unit length of pile is defined as the ratio of the static lateral force P_1 per unit length of pile at a depth z divided by the lateral displacement y at the same depth ($k = P_1/y$). The units of k are N/m^2 for example. The viscous constant c is defined as the ratio of the viscous lateral force P_2 per unit length of pile at a depth z over the lateral velocity $\delta y/\delta t$ at the same depth [$c = P_2/(\delta y/\delta t)$]. The units of c are N·s/m^2 for example.

The pile is modeled as an elastic member with a mass m per unit length of pile (kg/m, for example) and a bending stiffness EI; m and EI are constant with depth z and deflection y. The bending stiffness EI is defined as the ratio of the bending moment M existing at a depth z over the pile curvature $\delta^2y/\delta x^2$ at the same depth; EI is in N·m^2, for example. The lateral inertia force per unit length of pile is $P_3 = m\delta^2y/\delta t^2$.

The boundary conditions are that the pile has a length L, an embedded length also equal to L (no stick-up), that the shear and moment at the pile tip are zero at all times, that the moment at the pile top is zero at all times and that the shear at

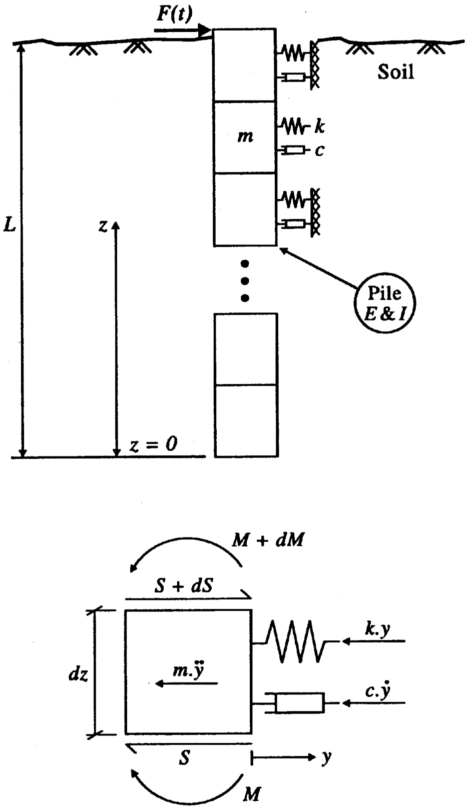

FIG. 3. Theoretical Model

the pile top is the forcing function $F(t)$. This forcing fun is represented by the impact force signal from the hamm the pile during the LATWAK test. Instead of solving this sient problem, the much simpler steady state vibration pro is solved where $F(t)$ is equal to $F_0e^{i\omega t}$ with ω being the an frequency. The reason for doing so is as follows.

The mobility is defined here as the modulus of the ra the frequency domain of the response velocity $v(t)$ at th cation of the applied force over the applied force $F(t)$. word modulus is used here in the sense of the modulus complex number. Indeed, as will be shown later, the ratio over $F(t)$ is a complex number. The mobility at the pile is given by the curve $|v/F|$ versus ω. For linear systems as the soil-pile system described above, the mobility is a p erty of the system and is independent of the applied force This principle is well explained in Ewins (1986). In a way mobility of a system in dynamics can be compared to stiffness of a system in statics; for linear systems the stiffness is independent of the applied force. This impo principle in dynamics is used to obtain the unique theore mobility curve of linear systems by solving the problem the simplest forcing function: the steady state vibration $F_0e^{i\omega t}$. This unique theoretical mobility curve can th matched with the experimental mobility curve even thoug experimental mobility is obtained from an impact force, steady state force. The assumption that, at the small involved in the impact, the soil-pile system behaves li is the basis and the limitation of the theory.

OBTAINING THEORETICAL MOBILITY CURVE

The governing differential equation for the dynamic be of the pile on Fig. 3 is (Ballouz and Briaud 1993)

$$EI\frac{\partial^4 y}{\partial z^4} + m\frac{\partial^2 y}{\partial t^2} + c\frac{\partial y}{\partial t} + ky = 0$$

This equation is obtained by making use of three fundam equations: the constitutive equation for the soil, the constitutive equation for the pile, and the dynamic equilibrium of the pile element. The boundary conditions are, for any time t

at $z = 0$ (pile tip), $\left.\frac{\partial^3 y}{\partial z^3}\right|_{z=0} = 0$ because the shear force is zero

$\left.\frac{\partial^2 y}{\partial z^2}\right|_{z=0} = 0$ because the moment is zero

at $z = L$ (pile top), $EI\left.\frac{\partial^3 y}{\partial z^3}\right|_{z=L} = -F_0 e\ i\omega t$ because the shear force is equal to $F(t)$

$\left.\frac{\partial^2 y}{\partial z^2}\right|_{z=L} = 0$ because the moment is zero

Knowing that the forcing function $F(t)$ is harmonic in time, and that, by assumption, the coefficients of the governing differential equations are constant, then the solution to (1) can be obtained by taking solutions of the form

$$y(z, t) = \theta e^{i(\omega t - Kz)} \tag{2}$$

where θ = displacement amplitude defining the envelope of pile movement, both with time and with depth (Fig. 4); ω = angular frequency; and K = complex wave number.

The complete derivation of the solution is in Ballouz and Briaud (1993). The solution of (1) with the boundary conditions listed is

$$y(z, t) = \frac{-2F_0 e^{i\omega t}}{EI\sigma^3\Delta}\{[\sin(\sigma L) - \sinh(\sigma L)][\cosh(\sigma z) + \cos(\sigma z)] + [\cosh(\sigma L) - \cos(\sigma L)][\sinh(\sigma z) + \sin(\sigma z)]\} \tag{3}$$

where

$$\sigma = \alpha e^{i\beta}; \quad \alpha = \left[\left(\frac{m\omega^2 - k}{EI}\right)^2 + \left(\frac{c\omega}{EI}\right)^2\right]^{1/8}$$

$$\beta = \frac{1}{4}\tan^{-1}\left(\frac{-c\omega}{m\omega^2 - k}\right) + \frac{\pi}{4}; \ \Delta = 2[1 - \cosh(\sigma L)\cos(\sigma L)]$$

In particular at the ground surface ($z = L$)

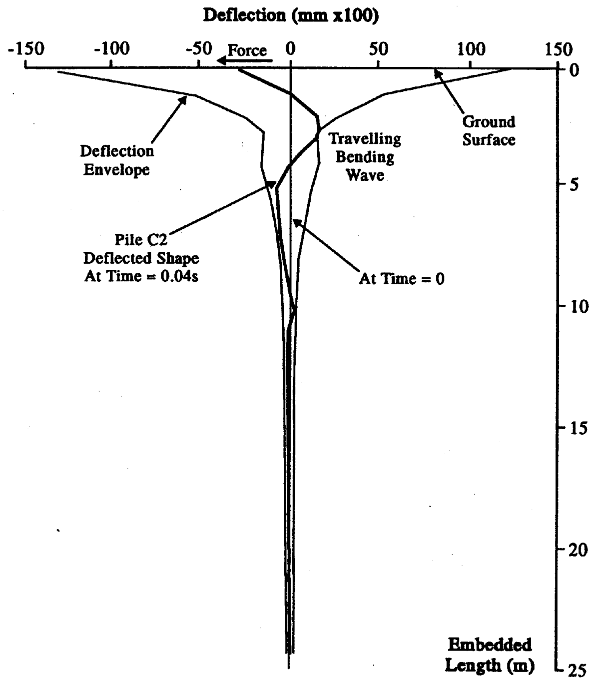

FIG. 4. Deflected Shape of Pile

$$y(L, t) = \frac{-2F_0 e^{i\omega t}}{EI\sigma^3\Delta}[\sin(\sigma L)\cosh(\sigma L) - \sinh(\sigma L)\cos(\sigma L)] \tag{4}$$

The velocity at the pile head can be obtained by taking the derivative of (3) and evaluating it at ($z = L$)

$$v(L, t) = \left.\frac{\partial y(z, t)}{\partial t}\right|_{z=L} \tag{5}$$

The forcing function $F(t)$ at the ground surface is a complex function ($F(t) = F_0 e^{i\omega t}$); the velocity function $v(L, t)$ at the ground surface is also a complex function because $y(L, t)$ is. Therefore the ratio $v(L, t)$) over $F(t)$ is a complex number. The mobility function at the ground surface is the modulus of the complex valued ratio of velocity over force

$$\left|\left.\frac{v(z, t)}{F(t)}\right|_{z=L}\right| = \left|\frac{-\omega((Re5 + Re6)^2 + (Im5 + Im6)^2)^{1/2}}{2EI\alpha^3((1 - a\bar{a} - b\bar{b})^2 + (a\bar{b} - b\bar{a})^2)^{1/2}}\right| \tag{6}$$

where

$Re5 = Re1 \cdot Re2 - Im1 \cdot Im2$

$Im5 = Re1 \cdot Im2 + Im1 \cdot Re2$

$Re6 = Re3 \cdot Re4 - Im3 \cdot Im4$

$Im6 = Re3 \cdot Im4 + Im3 \cdot Re4$

$Re1 = \sin(L\alpha\cos\beta)\cdot\cosh(L\alpha\sin\beta) - \sinh(L\alpha\cos\beta)\cdot\cos(L\alpha\sin\beta)$

$Re2 = \cosh(z\alpha\cos\beta)\cdot\cos(z\alpha\sin\beta) + \cos(z\alpha\cos\beta)\cdot\cosh(z\alpha\sin\beta)$

$Re3 = \cosh(L\alpha\cos\beta)\cdot\cos(L\alpha\sin\beta) - \cos(L\alpha\cos\beta)\cdot\cosh(L\alpha\sin\beta)$

$Re4 = \sin(z\alpha\cos\beta)\cdot\cosh(z\alpha\sin\beta) + \sinh(z\alpha\cos\beta)\cdot\cos(z\alpha\sin\beta)$

$Im1 = \cos(L\alpha\cos\beta)\cdot\sinh(L\alpha\sin\beta) - \cosh(L\alpha\cos\beta)\cdot\sin(L\alpha\sin\beta)$

$Im2 = \sinh(z\alpha\cos\beta)\cdot\sin(z\alpha\sin\beta) - \sin(z\alpha\cos\beta)\cdot\cos(z\ \alpha\sin\beta)$

$Im3 = \sinh(L\alpha\cos\beta)\cdot\sin(L\alpha\sin\beta) + \sin(L\alpha\cos\beta)\cdot\sinh(L\alpha\sin\beta)$

$Im4 = \cos(z\alpha\cos\beta)\cdot\sinh(z\alpha\sin\beta) + \cosh(z\alpha\cos\beta)\cdot\sin(z\alpha\sin\beta)$

$a = \cosh(L\alpha\cos\beta)\cdot\cos(L\alpha\sin\beta)$

$b = \sinh(L\alpha\cos\beta)\cdot\sin(L\alpha\sin\beta)$

$\bar{a} = \cos(L\alpha\cos\beta)\cdot\cosh(L\alpha\sin\beta)$

$\bar{b} = \sin(L\alpha\cos\beta)\cdot\sinh(L\alpha\sin\beta)$

$$\alpha = \left[\left(\frac{m\omega^2 - k}{EI}\right)^2 + \left(\frac{c\omega}{EI}\right)^2\right]^{1/8}$$

$$\beta = \frac{1}{4}\tan^{-1}\left(\frac{-c\omega}{m\omega^2 - k}\right) + \frac{\pi}{4}$$

The theortetical mobility function (6) is independent of time but is a function of the frequency ω. The LATWAK computer program was written to give, among many other things, a graphical display of the mobility function and of the pile deflection as a function of depth and time for the case of the steady state vibration. An example of pile deflection is shown in Fig. 4.

OBTAINING EXPERIMENTAL MOBILITY CURVE

The experimental mobililty curve is obtained by performing a LATWAK test in the field on the pile to be analyzed. The data gathered in a LATWAK test is in the time domain as in Fig. 2; it consists of the lateral force-time signal as given by the hammer tip load cell and the lateral velocity-time signal as given by the horizontal geophone glued to the pile. Such data is collected for each blow. The average signal for five blows is used in order to reduce the random noise which may exist in each blow.

The experimental mobility curve is conveniently obtained by the Discrete Fourier Transform technique (Bendat and Piersol 1986; Clough and Penzien 1993; Ballouz and Briaud 1993). Fourier showed in 1822 (Bringham 1974) that any function can be described as a superposition of sine and cosine functions of varying frequency. This mathematical technique is used to extract the frequency content of any function which exists in the time domain. For the function $F(t)$ and $v(t)$ in the time domain the following transformation gives the corresponding complex functions $F_{(\omega)}$ and $v_{(\omega)}$ in the frequency domain

$$F_{\omega_l} = \Delta t \left[\sum_{j=0}^{n-1} F_{tj} \cos\left(-2\pi \frac{lj}{n}\right) + i \sum_{j=0}^{n-1} F_{tj} \sin\left(-2\pi \frac{lj}{n}\right) \right]$$

for $l = 0, 1, 2 \cdots, n - 1$ (7)

and

$$v_{\omega_l} = \Delta t \left[\sum_{j=0}^{n-1} v_{tj} \cos\left(-2\pi \frac{lj}{n}\right) + i \sum_{j=0}^{n-1} v_{tj} \sin\left(-2\pi \frac{lj}{n}\right) \right]$$

for $l = 0, 1, 2 \cdots, n - 1$ (8)

where Δt = time elapsed between two consecutive readings by data acquisition system (say one-thousandth of second); n = number of data points collected during one blow in the LATWAK test (say 1,000); j = counter that gives time $j\Delta t$ in time domain signals; F_{tj} and v_{tj} = values of $F(t)$ and $v(t)$ in time domain for the jth time increment in the signals (values at $j\Delta t$); l = counter that gives value of $\omega_l = 2\pi(l/n) \times 1/\Delta t$; i = imaginary number ($\sqrt{-1}$); and F_{ω_l} and v_{ω_l} = values of F_ω and v_ω in frequency domain for value of ω equal to ω_l. Therefore the modulus of the complex valued ratio $v(t)$ over $F(t)$ is given by

$$\left|\frac{v(z, t)}{F(t)}\right|_{\omega_l} = \left[\frac{\left[\sum_{j=0}^{n-1} v_{tj} \cos\left(-2\pi \frac{lj}{n}\right)\right]^2 + \left[\sum_{j=0}^{n-1} v_{tj} \sin\left(-2\pi \frac{lj}{n}\right)\right]^2}{\left[\sum_{j=0}^{n-1} F_{tj} \cos\left(-2\pi \frac{lj}{n}\right)\right]^2 + \left[\sum_{j=0}^{n-1} F_{tj} \sin\left(-2\pi \frac{lj}{n}\right)\right]^2} \right]^{1/2}$$

for $l = 0, 1, 2 \cdots, n - 1$ (9)

The experimental mobility function is given in (9) and is calculated numerically in a discrete fashion by using the time domain data.

MATCHING EXPERIMENTAL AND THEORETICAL MOBILITY TO GET *k*

The stiffness k is obtained by matching the experimental mobility curve (9) and the theoretical mobility curve (6). Note that the experimental curve is obtained from an impact test while the theoretical curve is obtained from solving the problem of a steady state test. The uniqueness principle (Ewins 1986) states that for a linear system the mobility curve is unique and independent of the loading mode. It is assumed that, at the small strains involved in the LATWAK test, the soil-pile system in the experiment and in the theory is linear. Therefore the experimental mobility curve is the same as the theoretical curve even though they are obtained as responses to different loading modes and the matching process is valid.

The theoretical mobility curve depends on three variables: the stiffness of the soil spring k, the vibrating mass of pile plus soil m, and the damping coefficient of the system c. Those three variables are defined per unit length of pile. Trial and error to optimize the fit between the two curves would be time consuming and inefficient. Instead the system identification technique (Ljung 1987; Stubbs et al. 1994; and Glaser 1995) was used as follows. If $|v/F|^*_{\bar\omega}$ is the experimental curve (Fig. 5) and $|vF|^{(m,k,c)}_{\bar\omega}$ is the theoretical curve, the vertical distance between the two curves at a frequency $\bar\omega$ and for an initial guess m_0, k_0, c_0 is

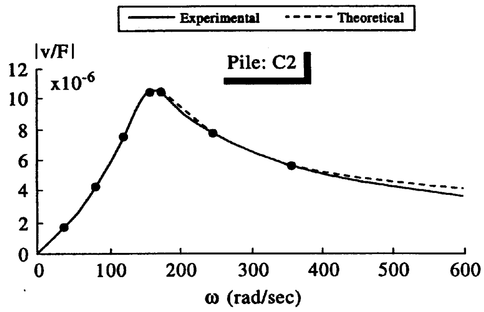

FIG. 5. Example of Curve Matching

$$\partial \left|\frac{v}{F}\right|_{\bar\omega} = \left|\frac{v}{F}\right|^*_{\bar\omega} - \left|\frac{v}{F}\right|^{(m_0,\, k_0,\, c_0)}_{\bar\omega} \quad (10)$$

This can also be calculated as

$$\partial \left|\frac{v}{F}\right|_{\bar\omega} = \left[\left(\frac{\partial \left|\frac{v}{F}\right|_\omega}{\partial m} \right)_{\bar\omega} \partial m + \left(\frac{\partial \left|\frac{v}{F}\right|_\omega}{\partial k} \right)_{\bar\omega} \partial k + \left(\frac{\partial \left|\frac{v}{F}\right|_\omega}{\partial c} \right)_{\bar\omega} \partial c \right]^{(m_0, k_0, c_0)} \quad (11)$$

The matching process consists of matching the two curves at n points until an acceptable precision is reached. In the present study it was found that matching the two curves at seven points, until the relative differences for all parameters $\partial m/m_0$, $\partial k/k_0$, and $\partial c/c_0$ were less than 2% consistently gave a good fit. For each of the seven points chosen, (11) is written. This leads to the following matrix equation

$$\underset{(7 \times 1)}{Z} = \underset{(7 \times 3)}{\psi} \cdot \underset{(3 \times 1)}{\xi} \quad (12)$$

where Z = matrix of the $\partial|v|/|F|_{\bar\omega_i}$; ψ = matrix of

$$\left(\frac{\partial \left|\frac{v}{F}\right|_\omega}{\partial m} \right)^{m_0, k_0, c_0}_{\bar\omega, i} \cdots$$

and ξ = matrix of (∂m, ∂k, ∂c). The matrix Z is known from (10); matrix ψ is known because the seven chosen frequencies $\bar\omega_i$ are known as well as the initial guesses m_0, k_0, c_0. The unknowns in (12) are ∂m, ∂k, ∂c (matrix ξ) and the solution is

$$\underset{(3 \times 1)}{\xi} = \underset{(3 \times 7)}{[\psi^T} \cdot \underset{(7 \times 3)}{\psi]^{-1}} \cdot \underset{(3 \times 7)}{\psi^T} \cdot \underset{(7 \times 1)}{Z} \quad (13)$$

Note that (12) represents seven equations with three unknowns (∂m, ∂k, ∂c), which are unlikely to satisfy all seven equations exactly. Eq. (13) is the mathematical expression of the least square method that allows optimization of the values of the 3 variables ∂m, ∂k, ∂c to best satisfy the seven equations in (12) (Ljung 1987; Stubbs et al. 1994). Once the ξ matrix is obtained, the parameter values can be updated.

$$\text{new } m = m_0 + \partial m$$

$$\text{new } k = k_0 + \partial k$$

$$\text{new } c = c_0 + \partial c$$

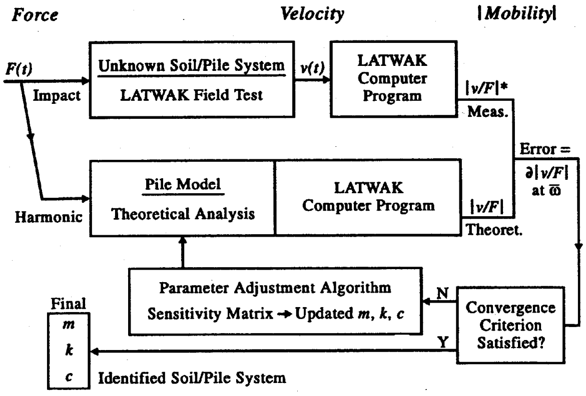

FIG. 6. Scheme for LATWAK Method

Eq. (13) is reentered with the updated values of m, k, and c. This leads to new adjustments ∂m, ∂k, ∂c. This process (Fig. 6) is repeated until the precision criterion of 2% is reached; that is to say, when the change in parameters is less than 2%. An example of curve matching is shown in Fig. 5.

The seven points used in the matching of the two curves are selected by the user of the program. The number 7 was obtained by trial and error as a compromise between the goodness of fit for the experimental and theoretical curves and the complexity of the calculations. Usually a couple of points are chosen close to the peak, two before and three after. Once the matching process has satisfied the 2% criterion, the user can view the two curves and accept the matching results or try a new set of seven points. Note that the final values of m, k, and c are not sensitive to the initial guesses for those parameters but the number of iterations to converge towards the 2% criterion is: the better the initial guesses, the shorter the convergence.

OBTAINING STATIC STIFFNESS *K*

The goal of the LATWAK test is to obtain the global lateral static stiffness K of the pile-soil system. This global stiffness K is the one that would be obtained at small deflections (say less than 1 mm) in a static lateral load test; it is the ratio of the lateral load over the lateral displacement. This global stiffness K is different from the soil spring stiffness k of the individual soil springs along the piles in the elementary model.

$$K = F_T/y_T \tag{14}$$

while

$$k = P/y \tag{15}$$

where F_T = horizontal load applied at pile top (kN); y_T = horizontal displacement at pile top (m); P = horizontal soil resistance per unit length of pile at depth z and deflection y (kN/m); and y = horizontal deflection of pile under P at depth z (m).

The solution to the static horizontal loading of a pile is a particular case of the dynamic problem solved earlier. The governing differential equation (1) becomes

$$EI\frac{\partial^4 y}{\partial z^4} + ky = 0 \tag{16}$$

because the damping and inertia terms are negligible in this case. The solution (Hetenyi 1946; Ballouz and Briaud 1993) reduces to

$$y(z) = \frac{F_T}{2EI\sigma^3 Q}\,[e^{-\sigma z}(P\cos(\sigma z) + R\sin(\sigma z)) + e^{\sigma z}((2R + P)\cos(\sigma z) + R\sin(\sigma z))] \tag{17}$$

where

$$\sigma = \sqrt[4]{\frac{k}{4EI}}; \quad R = \sin(\sigma L)(1 - e^{2\sigma L})$$

$$P = \cos(\sigma L) + 2e^{2\sigma L}\sin(\sigma L) - e^{2\sigma L}\cos(\sigma L)$$

$$Q = 2e^{\sigma L} + 4e^{\sigma z}\sin^2(\sigma L) - e^{3\sigma L} - e^{-\sigma L}$$

Eq. (17) gives the deflection y at any depth z of a pile which has a depth of embedment L, a bending stiffness EI and is loaded horizontally with a static force F_T applied at the ground surface in a soil with a soil spring stiffness k. In particular the pile deflection at the ground surface y_T is

$$y_T = \frac{F_T(e^{-\sigma L} + 4e^{\sigma L}\sin(\sigma L)\cos(\sigma L) - e^{3\sigma L})}{2EI\sigma^3 Q} \tag{18}$$

The lateral static stiffness K is therefore given by

$$K = \frac{F_T}{y_T} = \frac{k}{2\sigma}\,\frac{2e^{\sigma L} + 4e^{\sigma L}\sin^2(\sigma L) - e^{3\sigma L} - e^{-\sigma L}}{e^{-\sigma L} + 4e^{\sigma L}\sin(\sigma L)\cos(\sigma L) - e^{3\sigma L}} \tag{19}$$

Use (19) to calculate the global stiffness K once the soil spring stiffness k has been determined through the mobility curve matching process. The global stiffness K is the static stiffness predicted by the LATWAK test and is called K_p. It is to be compared to the global stiffness K_m measured in static lateral load tests to assess the potential of the LATWAK method.

A note of interest is that (16) is usually solved for two extreme cases: flexible pile ($L = \infty$) and rigid pile ($y = Rz + S$). The application of these solutions is restricted to the conditions $L > 3\sigma^{-1}$ for flexible piles and $L < \sigma^{-1}$ for rigid piles (Briaud 1992). In (18) the general solution for a pile of length L is given and can be used to evaluate the two conditions.

COMPARING *K* (LATWAK) WITH *K* (LOAD TEST)

A total of 20 lateral pile load tests were performed in conjunction with 20 LATWAK tests. The tests took place at three sites: Edmonton (Alberta, Canada), New Orleans, and Texas A&M University. There were eight driven piles in Edmonton, eight driven piles in New Orleans, and four bored piles at Texas A&M University. The characteristics of the piles are given in Table 1.

The soil in Edmonton consists of 5 m of compacted clayey fill, 3 m of fine silty sand, 12 m of soft clay, 1 m of glacial till and then dense silty sand. Detailed soil properties can be found in Briaud et al. (1994). The soil in New Orleans consists of 2 m of loose sand fill underlain by soft clay with intermittent layers of silty sand. Detailed soil properties can be found in Briaud et al. (1994). The two Texas A&M University sites are among the five National Geotechnical Experimentation Sites (NGES) sponsored by the Federal Highway Administration (FHWA) and the National Science Foundation (NSF). The soil at the clay site is a stiff clay, while the soil at the sand site is a medium dense sand. Detailed soil properties can be found in Briaud (1993) and Marcontell and Briaud (1994).

The lateral load tests were performed by applying the horizontal load in small increments and holding each load for 15 min while taking readings of displacements. The 15 min readings were used to generate the load-displacement curve. The measured stiffness value K_m was obtained from that curve by plotting the secant stiffness as a function of displacement and by extrapolating linearly to zero displacement the first two points on the curve (Fig. 7). Therefore K_m is an initial tangent stiffness obtained from the static load tests by extrapolation from data points corresponding to deflections of the order of 1 to 3 mm. The LATWAK tests were performed before the load tests. A typical case of LATWAK force-time and velocity-time results is shown in Fig. 2. Fig. 5 shows the corresponding results of the mobility curve matching process while Fig. 7